



Thai, N. L., Wu, X., Na, J., Guo, Y., Trung Tin, N. T., & Le, P. X. (2017). Adaptive variable step-size neural controller for nonlinear feedback active noise control systems. *Applied Acoustics*, 116, 337-347. <https://doi.org/10.1016/j.apacoust.2016.09.022>

Peer reviewed version

License (if available):  
CC BY-NC-ND

Link to published version (if available):  
[10.1016/j.apacoust.2016.09.022](https://doi.org/10.1016/j.apacoust.2016.09.022)

[Link to publication record in Explore Bristol Research](#)  
PDF-document

This is the author accepted manuscript (AAM). The final published version (version of record) is available online via Elsevier at <http://www.sciencedirect.com/science/article/pii/S0003682X16303000>. Please refer to any applicable terms of use of the publisher.

## University of Bristol - Explore Bristol Research

### General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:  
<http://www.bristol.ac.uk/red/research-policy/pure/user-guides/ebr-terms/>

# Adaptive Variable Step-Size Neural Controller for Nonlinear Feedback Active Noise Control Systems

Nguyen Le Thai<sup>1,2</sup>, Xing Wu<sup>1</sup>, Jing Na<sup>1,4\*</sup>, Yu Guo<sup>1</sup>, N.T. Trung Tin<sup>2</sup>, Phan Xuan Le<sup>3</sup>

1. Faculty of Mechanical & Electrical Engineering, Kunming University of Science & Technology, Kunming 650500, P.R. China

2. Faculty of Electric and Electronic Engineering, Tuy Hoa Industrial College, Tuy Hoa 620900, Vietnam

3. Faculty of Mechanical Technology, HCMC University of Food Industry, Ho Chi Minh 700000, Vietnam

4. Department of Mechanical Engineering, University of Bristol, Bristol BS8 1TR, UK

**Abstract-** Adaptive filter techniques and the **filtered-x least mean square (FxLMS)** algorithm have been used in Active Noise Control (ANC) systems. However, their effectiveness may degrade due to the nonlinearities and modeling errors in the system. In this paper, a new feedback ANC system with an adaptive neural controller and variable step-size learning parameters (VSSP) is proposed to improve the performance. A nonlinear adaptive controller with the FXLMS algorithm is first designed to replace the traditional adaptive FIR filter; then, a variable step-size learning method is developed for online updating the controller parameters. The proposed control is implemented without any offline learning phase, while faster convergence and better noise elimination can be achieved. The main contribution is that we show how to analyze the stability of the proposed closed-loop ANC systems, and prove the convergence of the presented adaptations. Moreover, the computational complexities of different methods are compared. Comparative simulation results demonstrate the validity of the proposed methods for attenuating different noise sources transferred via nonlinear paths, and the improved performance over classical methods.

**Keywords-** Active noise control, filtered-x least-mean-square (FxLMS) method, variable step-size learning, neural network, nonlinear path.

## 1. Introduction

Noise and vibration in the environment have drawn increasing attentions due to the use of various equipments that also serve as noise sources, such as motors, fans, transformers, etc. As reported by the World Health Organization [1, 2], noise and vibration cause a sensation of discomfort and harm to human health. Thus, the issue of noise control has been the focus of many researchers in the past decades. In general, there are two methods of noise control schemes, e.g. passive noise control (PNC) [3] and active noise control (ANC) [4]. The passive methods are effective for controlling the broadband noise and have been successfully used in the high-frequency noise circumstances. However, for narrowband noise and low-frequency noise, PNC is inefficient because of its bulky structure. Hence, ANC technology [5] was developed to handle more complex noise.

In general, ANC systems are designed based on the principle that a secondary noise source is generated to cancel the aggregate noise in the noise control area, which has a same amplitude but opposite phase compared to the primary noise. Recently, with advanced digital signal processing techniques, it is possible to obtain better acoustic performance by employing appropriate control approaches [6]. For this purpose, various ANC methods have been used in practical applications [5-9], e.g. vehicle transportation, aircraft engines, heating, air conditioning duct systems, etc. These available control methods are briefly classified into two categories: feedforward control [10] and feedback control [11]. Feedforward ANC systems must anticipate the reference signal input (noise source). Therefore, feedforward ANC approaches may not be feasible for the cases, where the noise source cannot be measured in the specific environments

---

This work was supported by National Natural Science Foundation (NSFC) of China under Grants 61573174, 51365023 and 51265018, and Basic Research Planning Project of Yunnan Province under Grant 2013FB028.

\*Corresponding author: Jing Na (najing25@163.com).

(e.g. high temperature, corrosive chemicals). To solve this problem, feedback ANC system (FANC) was developed, which uses only an error **signal in** the summing junction. Thus, feedback ANC strategies can address the unforeseen noise sources [12, 13]. Some available results show that feedback ANC systems based on adaptive finite impulse response (FIR) filter and **filtered-x least-mean-square (FxLMS)** algorithm work well for the cases with linear paths [14, 15]. However, in practical applications, the nonlinearities in the primary path and secondary path may degrade the control performance [16]. Thus, the development of nonlinear controllers/filters is necessary. In this respect, functional link neural networks (FLNN) were used in [13, 17] as feedback adaptive filters to cope with nonlinear systems. Adaptive bilinear filter [18][35] and adaptive recursive second-order Volterra (RSOV) filter [19] have been introduced for active control of nonlinear noise processes. Moreover, multilayer neural networks [20] were also used to control nonlinear plants. One notable issue in the **neural network (NN)** based ANC is the lack of appropriate fast learning algorithm and the rigorous proof of closed-loop stability.

Historically, the least mean square (LMS) algorithm has been used for ANC systems [21-24] due to their simplicity in the design and implementation. However, the convergence rate of conventional FXLMS algorithms may be **unsatisfactory** because the adopted constant learning gains (or step-size) may not be able to handle the wider operation regimes. To improve the convergence speed, several modifications have been introduced in [25-28]. The adjustment of the step-size parameter in [25] is implemented by minimizing the square of the control error. The simplicity of this algorithm allows us to implement it in practical ANC systems. However, the convergence of the time-varying step-size (VSS) parameter was not studied, and specific bounds of those learning gains should be selected in a **priori** manner to avoid the windup of the step-size, in particular when there are measurement noise. **Thus, an** extra time average of the error variables was further incorporated in [26] to retain its performance in the presence of measurement noise. In the recent work [27, 28], novel leakage terms have been imposed on the gradient-based algorithms to update the step-size parameters [27] and to adjust the tap length [28]. Although better performance can be obtained in comparison to [25], the increased computational costs and more stringent assumptions (e.g. independence condition) are required. Nevertheless, only linear feedforward ANC systems are studied via the FXLMS algorithm.

Based on the above observations, this paper presents a nonlinear neural network NN filter for ANC feedback systems, where an improved LMS algorithm with time-varying learning gains (or step-size parameter) is proposed to eliminate noise passing through the nonlinear path. This scheme can ensure the reliability and reduce the complexity of nonlinear feedback ANC systems. To online determine the NN weights with fast convergence, we suggest a new **variable step-size least mean square (VSS-LMS)** algorithm for varying step-size, where the idea initially proposed in [25] is modified to update the NN weights. The convergence of these step-size parameters are proved, where the Lyapunov function is used. In this sense, this paper introduces a new theoretical framework to prove the closed-loop stability and the convergence of the suggested adaptive laws without using the maximum and minimum bounds [26]. Finally, extensive comparisons and simulations are investigated to validate the suggested algorithms in terms of the computational complexity and the control performance.

This paper is organized as follows. Section 2 introduces the basis of FANC system, and the design of nonlinear FANC using a new VSS-LMS algorithm. In Section 3, appropriate comparisons to available results are studied to address the computational complexities. Simulations are shown in Section 4 and conclusions are summarized in Section 5.

## 2. Adaptive neural network feedback ANC system

### 2.1 Basis of feedback ANC system

The schematic of a feedback ANC system with adaptive control is presented in Figure 1 [29, 30], where  $d(n)$  is the noise generated by the noise source  $s(n)$  and propagated through the primary path  $P(z)$ , which should be eliminated.

The filter  $W(z)$  is used as the controller that generates the driving signal for the secondary noise source  $y(n)$  propagating through the secondary path  $S(z)$  to provide the canceling signal  $v(n)$  in the control noise area. A microphone is placed in this region to measure the residual noise  $e(n)$ . The control  $W(z)$  is online updated adaptively such that the residual noise  $e(n)$  can be minimized. To address this problem, the control parameters should be updated adaptively based on the reference  $\hat{d}(n)$  and the error  $e(n)$  because the exact noise  $d(n)$  cannot be measured.

In this system, the secondary path  $S(z)$  includes the dynamics of the digital-analog converter, analog-digital converter, reconstruction filter, power amplifier, loudspeaker, error microphone, preamplifier, acoustic path from the loudspeaker to the summing junction and the acoustic path from the summing junction to the error microphone, etc. In this paper, we assume that the secondary path  $S(z)$  is known. In fact,  $S(z)$  can be modeled by using a FIR filter (or NN)  $\hat{S}(z)$ , where its coefficients can be obtained in terms of off-line system identification methods. In particular, the modelling uncertainties will be addressed via the suggested nonlinear neural control.

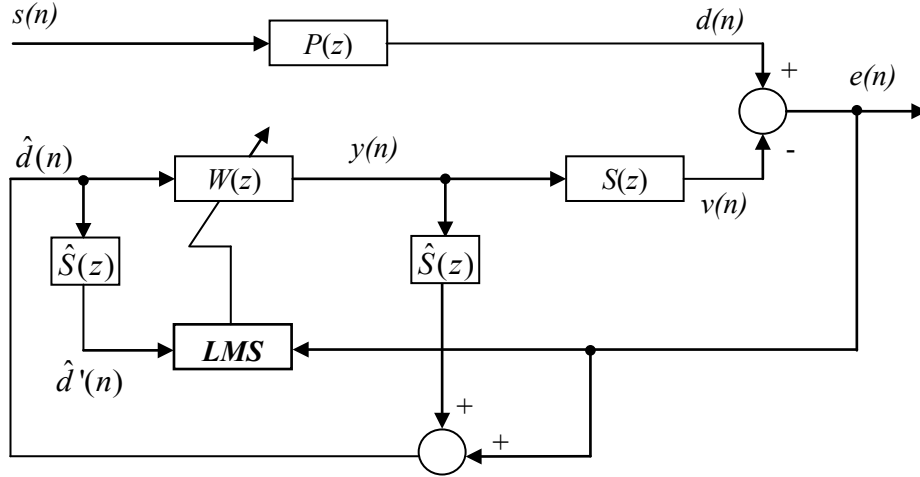


Figure 1. Schematic of feedback ANC system.

For the purpose of control design, various control schemes can be adopted as  $W(z)$ , e.g. FIR filter. However, in those linear controls (e.g. FIR filter), the model uncertainties of  $S(z)$  and the induced nonlinearities cannot be handled effectively. In this paper, a nonlinear control scheme with neural network will be introduced.

## 2.2 Feedback ANC System Design with LMS algorithm

This subsection will first present the idea for incorporating neural network into the ANC synthesis, and specifically, the closed-loop stability will be theoretically proved. The idea for using neural network is mainly motivated by the fact that neural network can be used to approximate unknown nonlinear smooth functions [31, 32]. The block diagram of the proposed nonlinear ANC system is shown in Figure 2.

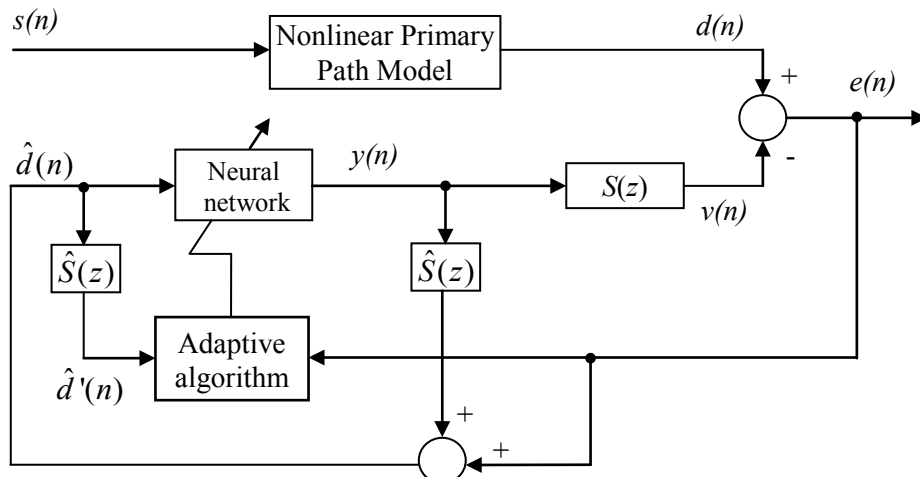


Figure 2. Adaptive neural network feedback ANC system.

The major difference between Figure 1 and Figure 2 lies in that a neural network is used as the nonlinear control  $W(z)$ . From Figure 2, the residual noise is given by

$$e(n) = d(n) - v(n) \quad (1)$$

with

$$v(n) = \sum_{j=0}^J s_j y(n-j) \quad (2)$$

$$\hat{d}(n) = e(n) + \sum_{j=0}^J \hat{s}_j y(n-j) \quad (3)$$

where  $s_j$  are the coefficients of the FIR filter  $S(z)$ , which define the dynamics of the secondary path dynamics,  $\hat{d}(n)$  is the reference signal serving as the input of ANC control, and  $\hat{s}_j$  is the estimation of  $s_j$ . Here, we assume  $\hat{s}_j = s_j$  because the model of the secondary path is assumed to be known. However, we do not need to measure the source noise.

The objective of this paper is to design a neural controller  $W(z)$  such that the residual noise  $e(n)$  can be regulated as small as possible. The detailed topology of neural network is shown in Figure 3, where  $w^1$  and  $w^2$  are the NN weights. Several neural networks, e.g. multi-layer perceptions, radial basis function networks, and recurrent neural networks (RNN), can be used. In this paper, **a NN with three-layers is designed as a non-linear filter**, where two adaptive algorithms are presented for determining the weights of the input hidden layer and the output layer, respectively.

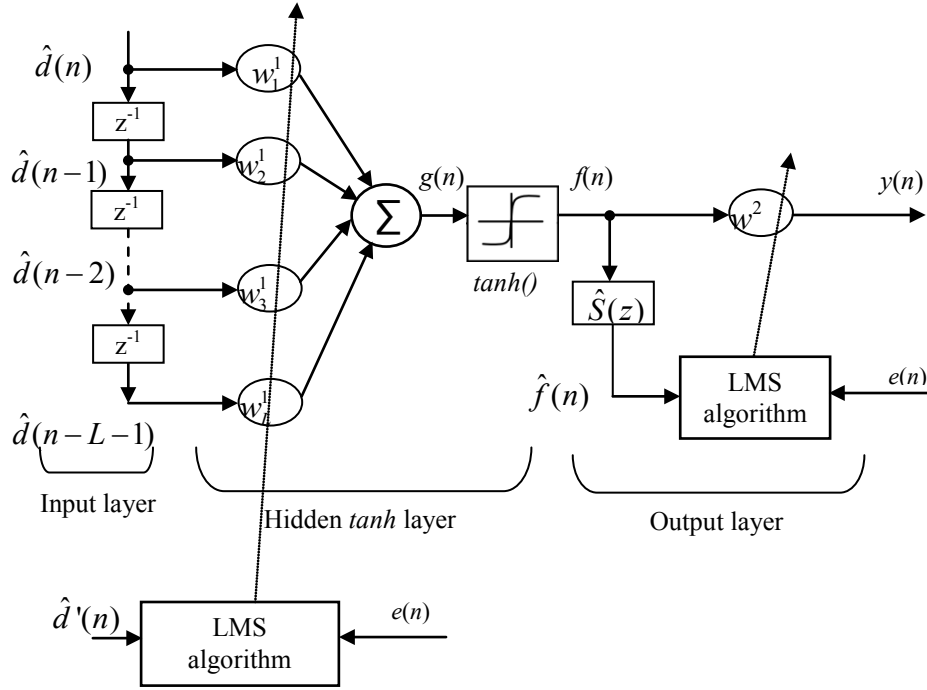


Figure 3. Diagram of adaptive neural controller using LMS algorithm.

In Figure 3, the layer 1 is the input layer that directly transmits the input signal to the hidden layer, the layer 2 is the hidden function layer, where activation functions  $\tanh(\bullet)$  are used, the layer 3 is the output layer to create the output. The dynamics of the used neural controller can be presented as

*Input Layer 1:*

$$a^{(1)}(n) = \hat{d}^{(1)}(n) \quad (4)$$

*Hidden Layer 2:*

$$a^{(2)}(n) = f(g(n)) \quad (5)$$

with

$$g(n) = \sum_{l=0}^L w_l^{(1)} \hat{d}(n-l) \quad (6)$$

$$f(n) = \tanh(g(n)) = \frac{e^{g(n)} - e^{-g(n)}}{e^{g(n)} + e^{-g(n)}} \quad (7)$$

Output Layer 3:

$$a^{(3)} = y(n) = w^{(2)}(n)f(n) \quad (8)$$

The remaining problem is to design appropriate adaptive laws such that the NN weights  $w^{(1)}(n), w^{(2)}(n)$  of the controller (4)-(8) can be online updated to minimize the residual error  $e(n)$ . For this purpose, we define a mean square cost function as  $\xi(n) = E[e^2(n)]$ , then the weights  $w^{(1)}(n), w^{(2)}(n)$  will be tuned to minimize the instantaneous error  $\xi(n) = e^2(n)$  according to the following descent gradient based adaptive law

$$w^i(n+1) = w^i(n) - \frac{\mu_i}{2} \nabla \xi^i(n), i = 1, 2 \quad (9)$$

where  $\mu_i > 0$  is the learning gain (step-size) of the proposed adaptation,  $\nabla$  is the differential operator defined by

$$\nabla \xi^i = \frac{\partial \xi}{\partial w^i} \quad (10)$$

Assume  $d(n)$  is independent of  $w(n)$  as [18], then by applying the chain rule to (10) based on (1)-(8), we have

$$\left[ \frac{\partial \xi(n)}{\partial w^{(1)}(n)} \right]^T = \frac{\partial \xi(n)}{\partial e(n)} \frac{\partial e(n)}{\partial v(n)} \sum_{j=0}^J \frac{\partial v(n)}{\partial y(n-j)} \frac{\partial y(n-j)}{\partial f(n-j)} \frac{\partial f(n-j)}{\partial g(n-j)} \left[ \frac{\partial g(n-j)}{\partial w^{(1)}(n)} \right]^T \quad (11)$$

where

$$\frac{\partial \xi(n)}{\partial e(n)} = 2e(n); \quad \frac{\partial e(n)}{\partial v(n)} = -1; \quad \frac{\partial v(n)}{\partial y(n-j)} = s_j; \quad \frac{\partial y(n-j)}{\partial f(n-j)} = w^{(2)}(n); \quad \left[ \frac{\partial g(n-j)}{\partial w^{(1)}(n)} \right]^T = \hat{d}(n-j) \quad (12)$$

$$\frac{\partial f(n-j)}{\partial g(n-j)} = \frac{(e^{(n-j)} + e^{-(n-j)})^2 - (e^{(n-j)} - e^{-(n-j)})^2}{(e^{(n-j)} + e^{-(n-j)})^2} = 1 - f^2(n-j) \quad (13)$$

From (11), (12) and (13), we have

$$\left[ \frac{\partial \xi(n)}{\partial w^{(1)}(n)} \right]^T = -e(n) \sum_{j=0}^J s_j [1 - f^2(n-j)] w^{(2)}(n) \hat{d}(n-j) \quad (14)$$

Thus, substituting (14) into (9)-(10), the weights of the hidden layer  $w^{(1)}(n)$  are updated by

$$w^{(1)}(n+1) = w^{(1)}(n) + \mu_1 e(n) \sum_{j=0}^J s_j [1 - f^2(n-j)] w^{(2)}(n) \hat{d}(n-j) \quad (15)$$

Consider the fact that

$$\hat{d}'(n) = \sum_{j=0}^J s_j \hat{d}(n-j) \quad (16)$$

Then we get the adaptive law for updating the coefficients  $w^{(1)}(n)$  as

$$w^{(1)}(n+1) = w^{(1)}(n) + \mu_1 w^{(2)}(n) e(n) \hat{d}'(n) [1 - f^2(n)] \quad (17)$$

Similarly, the updating law of the output layer weights  $w^{(2)}(n)$  can be deduced as

$$\left[ \frac{\partial \xi(n)}{\partial w^{(2)}(n)} \right]^T = \frac{\partial \xi(n)}{\partial e(n)} \frac{\partial e(n)}{\partial v(n)} \sum_{j=0}^J \frac{\partial v(n)}{\partial y(n-j)} \frac{\partial y(n-j)}{\partial w^{(2)}(n)} \quad (18)$$

$$\frac{\partial \xi(n)}{\partial e(n)} = 2e(n); \quad \frac{\partial e(n)}{\partial v(n)} = -1; \quad \frac{\partial v(n)}{\partial y(n-j)} = s_j; \quad \frac{\partial y(n-j)}{\partial w^{(2)}(n)} = f(n) \quad (19)$$

According to (9)-(10), the weights of  $w^{(2)}(n)$  are computed as

$$w^{(2)}(n+1) = w^{(2)}(n) + \mu_2 e(n) \sum_{j=0}^J s_j f(n-j) \quad (20)$$

Considering the fact  $\hat{s}_j = s_j$ , then the input to  $w^{(2)}(n)$  filter is derived by filtering the reference signal  $f(n)$  through the model  $\hat{S}(z)$  such that

$$\hat{f}(n) = \sum_{j=0}^J s_j f(n-j) \quad (21)$$

By substituting (21) into (20), we get the adaptive law for updating the coefficients  $w^{(2)}(n)$  as

$$w^{(2)}(n+1) = w^{(2)}(n) + \mu_2 e(n) \hat{f}(n) \quad (22)$$

The above adaptive laws (17) and (22) are derived based on the gradient descent method, thus the stability of the adaptive neural network feedback ANC system should be studied.

**Theorem 1:** For ANC system shown in Figure 2 with neural controller shown in Figure 3 and adaptive laws (17) and (22), if the learning gains fulfill  $0 < \mu_1 < 2 / \left\| \sum_{j=0}^J s_j [1 - f^2(n-j)] w^{(2)}(n) \hat{d}(n-j) \right\|^2$ ,  $0 < \mu_2 < 2 / \left\| \sum_{j=0}^J s_j f(n-j) \right\|^2$  then the closed-loop ANC system is stable.

**Proof:** The proof of Theorem 1 can be found in Appendix 1.

**Remark 1:** As shown in the proof of Theorem 1 (e.g. (48) and (49)), it can be concluded that the adaptive learning gains cannot be set arbitrarily large to retain the system stability. Moreover, the convergence performance of adaptive laws (17) and (22) depends on the step-size parameter  $\mu_i$ . In general, a large  $\mu_i$  may increase the convergence speed, while too large  $\mu_i$  can trigger instability of the overall system. Thus, constant learning gains may not be the best option to achieve satisfactory control performance.

### 2.3 Feedback ANC System Design with VSS-LMS algorithm

In the following, a variable step-size LMS algorithm will be presented for the NN control (4)-(8). The essential difference to the control shown in Section 2.2 is that the adaptive algorithm of NN output layer weights, i.e.  $w^{(2)}(n)$ , is replaced by the following scheme with time varying step-size parameters

$$w^{(2)}(n+1) = w^{(2)}(n) + \mu(n) e(n) \hat{f}(n) \quad (23)$$

where  $\mu(n)$  is a variable step-size parameter given as

$$\mu(n) = \begin{cases} \mu_{\max} & \text{if } \mu'(n) > \mu_{\max} \\ \mu'(n) & \text{otherwise} \end{cases} \quad (24)$$

where  $\mu_{\max}$  is the maximum allowable bound of the learning gain (this can be easily set as shown in Theorem 1), and  $\mu'(n)$  is the time-varying step-size parameter depending on the residual error  $e(n)$ , which will be updated by

$$\mu'(n+1) = \alpha \mu'(n) + \gamma e^2(n) \quad (25)$$

where the constant  $\alpha$  is a forgetting factor set as  $0 < \alpha < 1$ , and the constant  $\gamma > 0$  is the learning gain for the adaptation of  $\mu'$ , and the initial condition is set as  $\mu'(0) \geq 0$ .

A similar VSS-LMS algorithm can be derived for the weights  $w^{(1)}(n)$  for the hidden layer of NN control (4)-(8). However, to simplify the practical implementation and seek a trade-off between the performance and the computational costs, the LMS algorithm (17) with a constant gain  $\mu_1(n) = \text{const}$  is used.

**Remark 2:** As shown in (25), one can verify that  $\mu'(n) > 0$  holds for all  $n = 1, \dots, \infty$  with any initial condition  $\mu'(n) \geq 0$ , i.e. the step-size is always positive. Moreover,  $\mu'(n)$  is updated based on the control error  $e(n)$ . From (25),

it can be found that the step-size will increase for a large control error  $e(n)$ , which can **provide** faster convergence of  $w^{(2)}(n)$  and thus enhance the control performance. On contrary, when the control error  $e(n)$  is small, the step-size  $w^{(2)}(n)$  can be retained as constant to avoid the mis-adjustment. Moreover, the constant  $0 < \alpha < 1$  is used as a forgetting factor to guarantee the boundedness of the learning gain  $\mu'$ . In fact, we can prove that for sufficiently small  $\gamma > 0$ , the step-size  $\mu'$  will converge to a positive constant  $\mu^* > 0$  when  $e(n)$  is bounded, i.e.  $\mu' \rightarrow \mu^*$  for  $n \rightarrow \infty$ .

**Remark 3:** According to the property of NN approximation, for any given noise  $d$ , there exists an ideal optimal value  $w^{(2)*}$  of the NN weight  $w^{(2)}$  in (8) so that  $d(n) = w^{(2)*} \bar{y}(n)$  holds, where  $\bar{y}(n) = s(n) * y(n)$  is the signal generated by passing the reference signal through the secondary path, and the operator  $*$  denotes the linear convolution. It is worth noting that the ideal constants  $w^{(2)*}$  and gain  $\mu^*$  are used for the analysis only, thus they can be unknown because we do not use them in the control implementation. Finally, we will also prove that the NN weights  $w^{(2)}$  updated based on (23) will converge to the optimal value  $w^{(2)*}$ .

The above two claims will be summarized and proved in the following Theorem 2. To facilitate further analysis, we define the parameter estimation errors of the proposed adaptive laws (23) and (25) as  $\tilde{w}(n) = w^{(2)}(n) - w^{(2)*}$  and  $\tilde{\mu}'(n) = \mu'(n) - \mu^*$ , respectively. Then, by subtracting  $w^{(2)*}$  and  $\mu^*$  from both sides of (23) and (25), then it follows

$$\tilde{w}(n+1) = \tilde{w}(n) + \mu e(n) \sum_{j=0}^J s_j F(n-j) \quad (26)$$

where

$$F(n) = [f(n), f(n-1), f(n-J-1)] \quad (27)$$

On the other hand, considering (1), (2), (8) and above Remark 3, one may obtain that

$$e(n) = d(n) - v(n) = w^{(2)*T} [s(n) * y(n)] - w^{(2)T} [s(n) * y(n)] = -\tilde{w}^T(n) \bar{y}(n) = -\bar{y}^T(n) \tilde{w}(n) \quad (28)$$

Then substituting (28) into (26) will yield

$$\tilde{w}(n+1) = \tilde{w}(n) + \mu e(n) \sum_{j=0}^J s_j F(n-j) = \tilde{w}(n) - \mu R(n) \tilde{w}(n) \quad (29)$$

where  $R(n) = \bar{y}^T(n) \bar{y}(n)$  is a positive variable denoting the input vector correlations [33].

**Theorem 2:** For ANC system given in Figure 3, the controller (8) is applied with adaptive laws (17), (23) and (25), then for any  $0 < \alpha < 1$  and sufficiently small  $\gamma > 0$ , the NN weights  $w^{(2)}$  and the variable step-size  $\mu'$  converge to their ideal values  $w^*$  and  $\mu^*$ . Moreover, the closed-loop ANC system is stable.

**Proof.** The proof will be proposed in two steps.

1) In the first step, we prove  $w^{(2)}$  converges to  $w^*$ . Select a Lyapunov function as

$$V(n) = V_1(n) + V_2(n) = \frac{1}{2} e^2(n) + \frac{1}{2} \tilde{w}^T(n) \tilde{w}(n) \quad (30)$$

Then similar to (44) ~ (47) (as shown in Appendix), one can obtain the difference of  $V_1(n)$  as

$$\Delta V_1(n) = V_1(n+1) - V_1(n) = -\frac{1}{2} \mu e^2(n) R(n) [2 - \mu R(n)] \quad (31)$$

Moreover, we can calculate the difference of  $V_2(n)$  based on (26) and (28) as

$$\begin{aligned} \Delta V_2(n) &= V_2(n+1) - V_2(n) = \frac{1}{2} [\tilde{w}^T(n+1) \tilde{w}(n+1) - \tilde{w}^T(n) \tilde{w}(n)] \\ &= \frac{1}{2} \left\{ [\tilde{w}(n) + \mu e(n) \bar{y}(n)]^T [\tilde{w}(n) + \mu e(n) \bar{y}(n)] - \tilde{w}^T(n) \tilde{w}(n) \right\} \\ &= \mu e(n) \tilde{w}^T(n) \bar{y}(n) + \frac{1}{2} \mu^2 e^2(n) R(n) \\ &= \mu R(n) \tilde{w}^T(n) \tilde{w}(n) + \frac{1}{2} \mu^2 e^2(n) R(n) \end{aligned} \quad (32)$$



Thus, the difference of  $V(n)$  can be obtained as

$$\Delta V(n) = \Delta V_1(n) + \Delta V_2(n) = -\frac{1}{2}\mu e^2(n)R(n)\left[2 - \mu(1 + R(n))\right] - \mu R(n)\tilde{w}^T(n)\tilde{w}(n) \quad (33)$$

If we set  $0 < \mu < 2 / (1 + R(n))$ , which can be fulfilled by selecting the maximum allowable bound in (24) as  $\mu_{\max} \leq 2 / (1 + R(n))$ , one can verify  $\Delta V(n) < 0$  for all  $n > 0$ , and  $V(n+1) = V(n) + \Delta V(n) < V(n)$ , which implies  $V(n) \rightarrow 0$  for  $n \rightarrow \infty$ . Moreover, the fact  $V(n) \rightarrow 0$  further implies  $e(n) \rightarrow 0$  and  $\tilde{w}(n) \rightarrow 0$  for  $n \rightarrow \infty$ , such that  $w^{(2)} \rightarrow w^*$  holds for  $n \rightarrow \infty$ .

2) In the second part, the convergence of the learning gain  $\mu'$  to its ideal value  $\mu^*$  will be studied. In general ANC systems, there may be unavoidable modeling errors in the primary and secondary path models, so that the control error  $e(n)$  may only converge into a small bounded set around zero rather than converge to zero [34]. Without loss of generality, we assume that the control error is bounded as  $|e(n)| \leq \varepsilon$  for a small positive constant  $\varepsilon > 0$ . In this case, one can easily obtain from the adaptive law (25) with any initial condition  $\mu'(0) \geq 0$  that

$$\begin{aligned} \mu'(1) &= \alpha\mu'(0) + \gamma e^2(0) \\ \mu'(2) &= \alpha\mu'(1) + \gamma e^2(1) = \alpha^2\mu'(0) + \alpha\gamma e^2(0) + \gamma e^2(1) \\ &\vdots \\ \mu'(k+1) &= \alpha^{k+1}\mu'(0) + \alpha^k\gamma e^2(0) + \alpha^{k-1}\gamma e^2(1) + \cdots + \alpha\gamma e^2(k-1) + \gamma e^2(k) \\ &\vdots \end{aligned} \quad (34)$$

One can recall the property of geometric series  $\lim_{n \rightarrow \infty} \sum_{k=0}^{n-1} \alpha^k = \frac{1}{1-\alpha}$  and  $\lim_{n \rightarrow \infty} \alpha^n = 0$  for  $0 < \alpha < 1$ . Then, based on the fact that  $|e(n)| \leq \varepsilon, n = 0, 1, \dots, \infty$  and the initial condition  $\mu'(0) \geq 0$  is finite, it can be verified from (34) that

$$\mu'(\infty) = \lim_{n \rightarrow \infty} \mu'(n+1) \leq \lim_{x \rightarrow \infty} \alpha^{n+1}\mu'(0) + \lim_{x \rightarrow \infty} \sum_{n=0}^{\infty} \alpha^n \gamma \varepsilon = \frac{\gamma \varepsilon}{1-\alpha} \quad (35)$$

We can easily conclude from (35) that the variable step-size  $\mu'$  is bounded and converges to  $\mu^* = \frac{\gamma \varepsilon}{1-\alpha}$ . In this case, one can select parameters in (25) as  $0 < \alpha < 1$  and  $\gamma \leq \frac{1-\alpha}{\varepsilon R(n)}$  so that the condition  $0 < \mu < 2 / (1 + R(n))$  can be fulfilled for all  $n = 0, 1, \dots, \infty$ , and then based on Theorem 1, we can claim that the closed-loop system is stable.

**Remark 4:** The main contribution of this paper is to propose a new theoretical framework to study the stability of ANC system with variable step-size and to prove the convergence of these time-varying coefficients (e.g.  $w$  and  $\mu'$ ) to their ideal values. It is noted that in available variable step-size LMS algorithms (e.g. [25-28]), the convergence property of such adaptive parameters have rarely been addressed, and the ANC system stability with variable step-size is still not fully solved in the literature. Thus, the results of this paper may fill in this gap.

### 3. Comparisons

In order to show the efficiency of the proposed methods, this section will compare them with the classical linear ANC with the FXLMS algorithm (e.g.  $W(z)$  is a FIR filter as [11, 12]), and analyze the computational complexities of 1) a linear FIR filter **control with FxLMS** scheme (will be presented in Section 3.1); 2) nonlinear NN control (8) with the LMS algorithm (17) and (22) (as Section 2.2); 3) nonlinear NN control (8) with the VSS-LMS algorithm (17), (23) and (25) (as Section 2.3).

#### 3.1 Linear FIR control with FXLMS algorithm [11, 12]

In this case, an adaptive filter is used as the control, where the output of adaptive control  $W(z)$  can be written as

$$y(n) = \sum_{k=0}^{N-1} w_k(n)\hat{d}(n-k) = w^T(n)\hat{d}(n) \quad (36)$$

with

$$w(n) = [w_0(n), w_1(n), \dots, w_{N-1}(n)] \quad (37)$$

$$\hat{d}(n) = [\hat{d}(n), \hat{d}(n-1), \hat{d}(n-N+1)]^T \quad (38)$$

where  $N$  is the order of the filter,  $w(n)$  are the coefficients of the filter  $W(z)$ , which can be tuned to minimize the instantaneous error  $\xi(n) = e^2(n)$  according to the following gradient descent method

$$w(n+1) = w(n) - \frac{\mu}{2} \nabla \xi(n) \quad (39)$$

where  $\mu > 0$  is a constant. By applying mathematical manipulations similar to Section 2, one can get the adaptive law

$$w(n+1) = w(n) + \mu e(n) \sum_{j=0}^J s_j \hat{d}'(n-j) \quad (40)$$

where  $\hat{d}'(n) = \sum_{j=0}^J \hat{s}_j \hat{d}(n-j)$ .

Similar to the arguments of Section 2, the stability of the proposed control (36) with adaptive law (40) can be given as

**Theorem 3:** For ANC system shown in Figure 1 with **linear FIR control** (36) and adaptive law (40), if the learning gain fulfills  $0 < \mu < 2 / \left\| \sum_{j=0}^J s_j \hat{d}(n-j) \right\|^2$ , then the ANC system is stable.

The proof of Theorem 3 is similar to that of Theorem 1, and thus will not be repeated again.

### 3.2 Computational complexity analysis

In this subsection, the practical implementation of the proposed three controllers will be presented, and their computational complexities will be compared.

#### 1) Linear FIR filter control with FXLMS algorithm

The implementation of the FIR filter control (36) with FxLMS algorithm (40) can be given as:

- (1) **Calculating** the FIR filter outputs (36), which requires  $N$  multiplications and  $N-1$  additions, where  $N$  denotes the length of the adaptive FIR filter.
- (2) **Computing** the reference signal  $\hat{d}(n)$  based on (3), which needs  $J$  multiplications and  $J$  additions, where  $J$  is the length of the secondary path model  $\hat{S}(z)$  using a FIR filter.
- (3) **Updating** the filter weight coefficients by (40), which uses  $N+1$  multiplications and  $N$  additions.

Therefore, the linear FIR filter based ANC system needs  $2N + J + 1$  multiplications and  $2N + J - 1$  additions.

#### 2) Nonlinear neural network control with LMS algorithm

For fair comparison, we choose the dimension of the NN weights  $w^{(1)}(n)$  in the neural control (8) **are the same as that** in the FIR filter as  $N$ . Then, the computational complexity of neural control (8) with the LMS algorithm (17) and (22) is calculated as:

- (1) **Calculating** the NN hidden layer output as (6), which requires  $N$  multiplications and  $N-1$  additions.
- (2) **Calculating** the NN output as (7), which uses 1 subtraction, 1 addition, 1 division **and 4 exponentiations**.
- (3) **Calculating** the control output as (8), which needs 1 multiplication.
- (4) **Computing** the reference signal  $\hat{d}(n)$  based on (3), which requests  $J$  multiplications and  $J$  additions.
- (5) **Computing** the reference signal  $\hat{f}(n)$  based on (21), which requires  $J$  multiplications and  $J-1$  additions.
- (6) **Updating** the coefficients by (17), which employs 1 subtraction,  $N+4$  multiplications and  $N$  additions.
- (7) **Updating** the NN **weights** by (22), which uses 2 multiplications and 1 addition.

Therefore, the nonlinear neural controller ANC system with LMS algorithm needs  $2N + 2J + 7$  multiplications, 1 division, 2 subtractions,  $2N + 2J$  additions and 4 exponentiations.

### 3) Nonlinear neural network control with VSS-LMS algorithm

The implementation of the nonlinear neural control (8) with the VSS-LMS algorithm (17),(23) and (25) is similar to that of the above neural control with the LMS algorithm, **while** extra operations are required to calculate the variable step size  $\mu'$  based on (25). To update the variable step size by  $\mu'(n+1) = \alpha\mu'(n) + \gamma e^2(n)$ , 2 multiplication and 1 addition are needed. Therefore, the proposed nonlinear neural control with VSS-LMS algorithm imposes  $2N + 2J + 9$  multiplications, 1 division, 2 subtractions,  $2N + 2J + 1$  additions and **4 exponentiations**.

The computational costs of the above three control methods are summarized in Table 1. Here, we can see that their computational costs are mainly proportional to  $N$  and  $J$  (i.e. the order of control and secondary path). Moreover, the computational cost of the proposed method is almost the same as that of neural control with the LMS algorithm. However, the computational costs of the proposed nonlinear controls with neural network are more demanding compared to the linear FIR filter control. This is reasonable because a nonlinear NN is introduced as the controller to eliminate **nonlinear noise signals** and to accommodate the modelling uncertainties.

**Table 1. Comparisons of computational costs**

Controllers	Multiplications	Divisions	Additions	Subtractions	$\exp(\cdot)$
FIR, LMS	$2N + J + 1$	0	$2N + J - 1$	0	<b>0</b>
NN with LMS	$2N + 2J + 7$	1	$2N + 2J$	2	<b>4</b>
NN with VSS-LMS	$2N + 2J + 9$	1	$2N + 2J + 1$	2	<b>4</b>

### 4. Simulations

In this section, extensive simulation results are presented to compare the performances of different FANC systems with a nonlinear primary path. In the simulations, the sampling frequency is chosen to be 8 kHz, and a non-minimum phase secondary path **is used**, whose model is given as

$$v(n) = y(n-4) - 0.3y(n-5) + 0.2y(n-6) \quad (41)$$

We compare the performances of three different control approaches:

- (1) FANC system using linear filter control (36) with FXLMS method (40);
- (2) Neural network FANC **(8)** with LMS algorithm (17) and (22);
- (3) Neural network FANC **(8)** with VSS-LMS algorithm (17), (23) and (25).

**Case 1):** In this case, the primary path is selected a second-order nonlinear polynomial

$$d(n) = s(n-3) - 0.3s(n-4) + 0.2s(n-5) + 0.8s^2(n-5) \quad (42)$$

and the noise source  $s(n)$  is chosen as a sinusoidal signal with frequency of 200 Hz. To indicate the performance of the proposed method, we select parameters of the **FIR control with FxLMS method as  $N = 32$  and  $\mu = 0.001$ . The parameters of NN control (8) with LMS method (22) are set as  $N = 32$  and  $\mu_1 = 0.00001$ ,  $\mu_2 = 0.001$ . The parameters of NN control (8) with VSS-LMS (23) and (25) are selected as  $N = 32$  and  $\mu_1 = 0.00001$ , the initial step-size parameter as  $\mu'(0) = 0.0001$  and the forgetting factors as  $\alpha = 0.98$  and  $\gamma = 0.1$ . These parameters can be selected by applying a trial-and-error method and should be chosen as a trade-off between the stability and control performance. As pointed out in Theorem 1 and Theorem 3, to retain the closed-loop ANC system stability, the step-size of the above ANC methods (e.g. linear control with FxLMS, NN control with LMS and NN control with VSS-LMS) cannot be set very large. Thus, we can choose small initial values, and then increase them until the system indicates instability**

phenomenon. This leads to the selection of  $\mu = 0.001$  in the FxLMS algorithm. For fair comparisons in the simulations, although a larger step-size  $\mu_2$  of NN control (8) with LMS (22) may be allowed (because the regressor  $\hat{f}(n)$  in (22) is the output of sigmoidal function (7), and thus its amplitude is small, which in turn allows for a larger step size), we choose  $\mu_2 = 0.001$  for NN control (22), which is the same as  $\mu = 0.001$  used in the FxLMS algorithm.

Figure 4 shows the profile of the original noise and the residual noise signals with different control methods in the time-domain. From Figure 4, we can find that the linear FIR filter control is inefficient when the system has nonlinearities (e.g. primary and second paths). Nonlinear NN control with the LMS algorithm performs slightly better than linear FIR filter method, and the proposed NN control with VSS-LMS algorithm obtains best performance in terms of convergence performance. Figure 5 provides the simulation results of the canceling noise in the frequency domain. The residual power spectrums of linear FIR filter control with FxLMS, NN control with LMS and the proposed NN control with VSS-LMS are depicted in dot, dash and dash-dot lines, respectively, and the solid line shows the power spectrum of the residual noise signals, when the ANC scheme is turned off. From Figure 4 and Figure 5, we can conclude that the proposed nonlinear NN control with time-varying learning gain in this paper can provide improved control performance compared to other methods in both time-domain and frequency-domain. This is reasonable because the adopted NN can handle nonlinearities effectively, and the proposed time-varying learning gain for the NN weights can achieve faster convergence response. To show this fact, Figure 6 indicates the profile of  $\mu'$  with the proposed VSS-LMS method (25), which illustrates that this step-size converges to a constant as proved in Theorem 2. Moreover, the evolutions of the NN coefficient  $w^{(2)}$  with VSS-LMS are shown Figure 7, where the response of  $w^{(2)}$  with the VSS-LMS algorithm is shown in solid line, and with the LMS algorithm is given in the dash line. It can be clearly seen that faster adaptation of the VSS-LMS algorithm is achieved than LMS method.

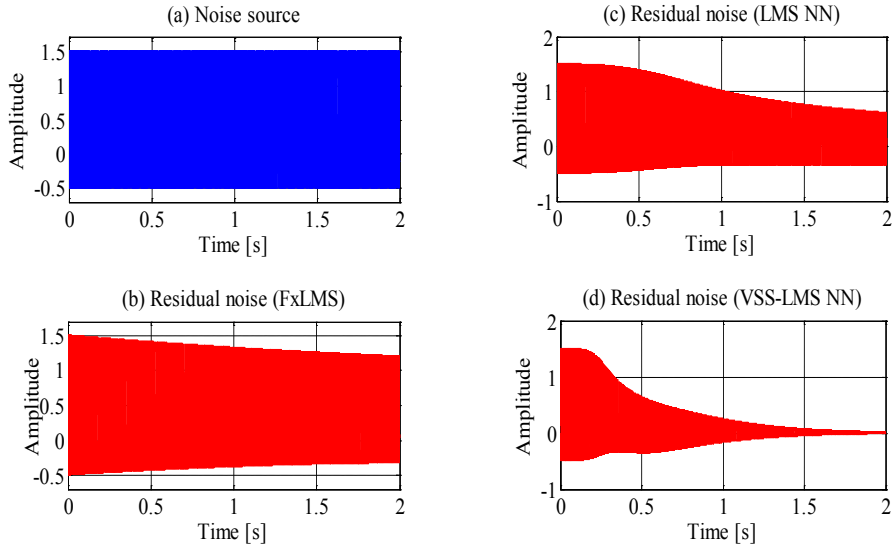


Figure 4. Simulation results for noise frequency 200 Hz in time domain. (a) Noise source; (b) Residual noise of linear control with FxLMS; (c) Residual noise of NN control with LMS; (d) Residual noise of NN control with VSS-LMS.

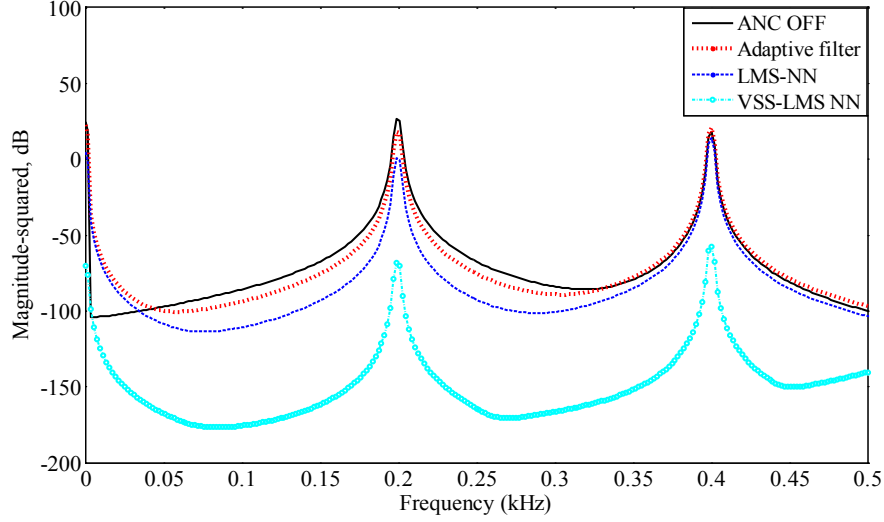


Figure 5. Simulation results for noise frequency 200Hz in frequency domain.

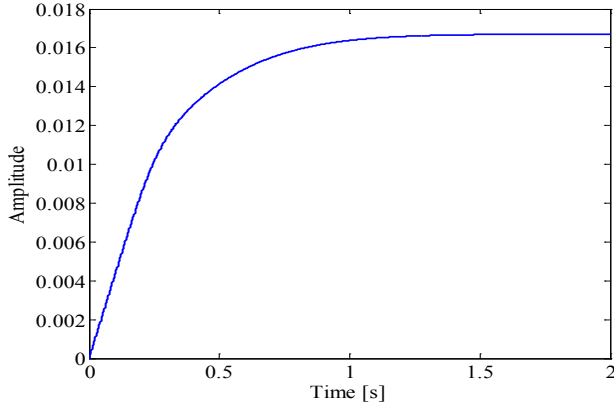


Figure 6. Convergence performance of  $\mu'$ .

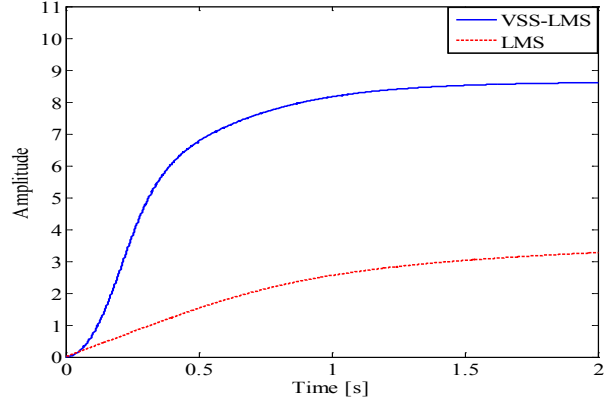


Figure 7. Profiles of  $w^{(2)}$  in NN control.

**Case 2):** To further show the ability of the proposed method to handle nonlinearities, the primary path is set as a third-order nonlinear polynomial as

$$d(n) = s(n-2) + 0.8s^2(n-2) - 0.4s^3(n-1) \quad (43)$$

and other system configurations are selected the same as Case 1. However, since both the primary and secondary paths are nonlinear, the order of linear FIR filter and the neural network are increased to  $N = 64$ .

Figure 8 depicts the noise source signal and the noise cancellation performance with three different control methods in the time-domain. Similar to **Case 1)**, it is also shown that the proposed NN control with the VSS-LMS algorithm can obtain best performance among the aforementioned three methods in terms of convergence speed and residual error. Figure 9 gives the simulation results in the frequency-domain, where the power spectrums of the residual error are provided. Figure 8 and Figure 9 also illustrate that the proposed FANC system can effectively attenuate the narrowband noises for highly nonlinear systems. It is also found that **more neurons in this case are** needed because both primary and secondary paths are nonlinear in contrast to Case 1). This needs more computational costs as shown in Section 3.2. Thus, a trade-off can be made between the performance and complexity in this case. Figure 10 indicates the response of  $\mu'$  for the proposed VSS-LMS method, and Figure 11 shows the NN coefficient  $w^{(2)}$  of two NN control methods, where the proposed VSS-LMS algorithm can achieve faster convergence rate, **which in turn contributes** to the improved control performance.

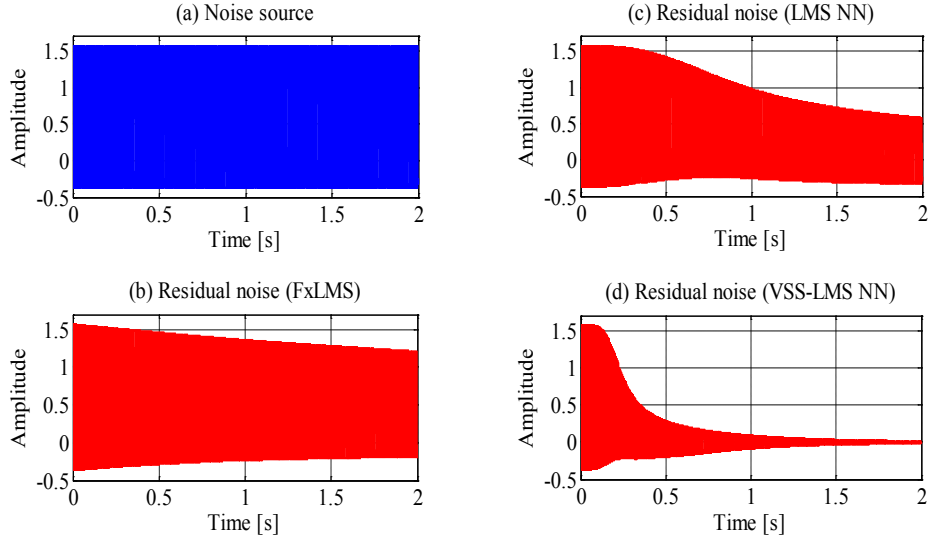


Figure 8. Simulation results for noise frequency 200 Hz in time domain. (a) Noise source; (b) Residual noise of linear control with FxLMS; (c) Residual noise of NN control with LMS; (d) Residual noise of NN control with VSS-LMS.

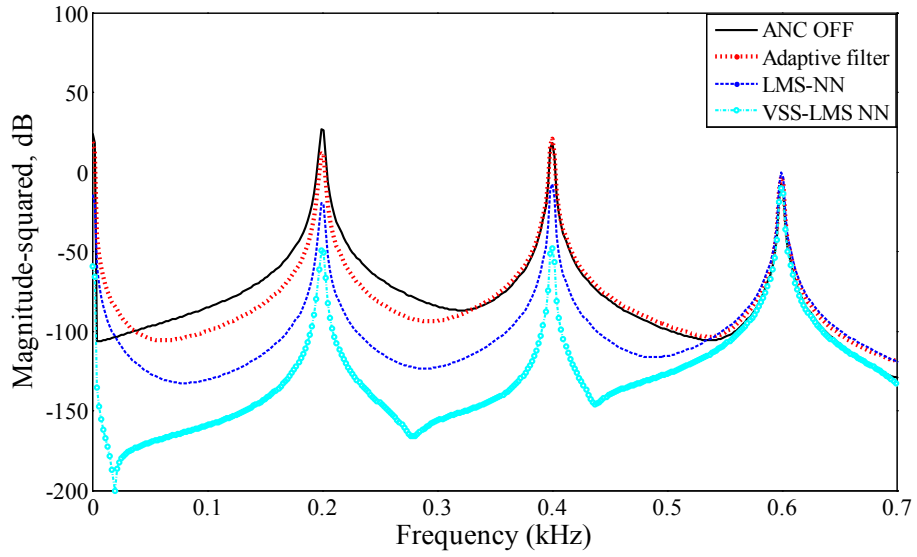


Figure 9. Simulation results for noise frequency 200 Hz in frequency domain

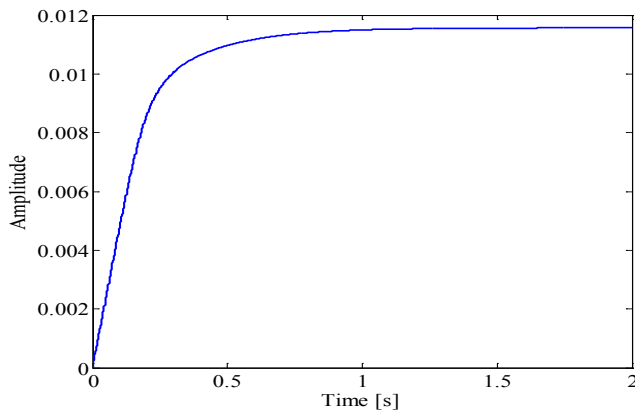


Figure 10. Convergence performance of  $\mu'$ .

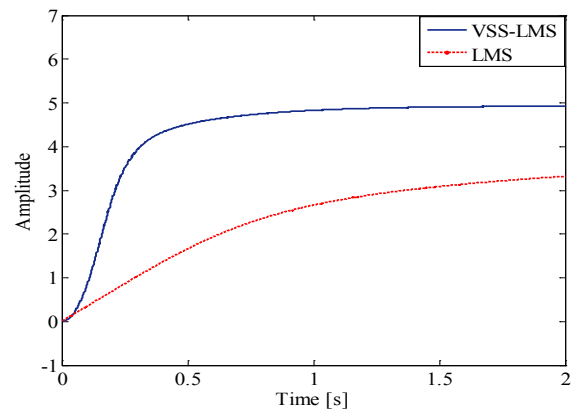


Figure 11. Profiles of  $w^{(2)}$  in NN control.

**Case 3):** In this case, we will show the control performance for more complex noise. Thus, we simulate the source noise as a multi-sinusoidal signal with two frequency components: 100Hz and 200Hz. The same primary and secondary

paths are used as Case 1). Simulations parameters of the ANC methods are selected as those used in the Case 2). Simulation results are shown in Figure 12 - Figure 15. Figure 12 depicts the noise signal and the noise cancellation performance of different control methods in the time-domain. This figure verifies that the FANC system using linear FIR control is not able to attenuate mutli-frequency noise, and the proposed nonlinear NN control with VSS-LMS algorithm performs better than the NN control with constant learning gains (e.g. LMS). This is reasonable because we use a varying step-size parameter to increase the convergence rate of the LMS algorithm. Figure 13 gives the simulation results in the frequency-domain, which also validates the observations in the time-domain. Thus, one can find that the performance of linear FIR control is worse than the proposed nonlinear NN control methods. Figure 14 depicts the convergence response of  $\mu'$  with the suggested updating law (25), which converges to a constant as claimed in Theorem 2. The profiles of NN weights  $w^{(2)}$  with constant and time-varying learning gains are shown in Figure 15, where faster convergence can be achieved by using the VSS-LMS algorithm.

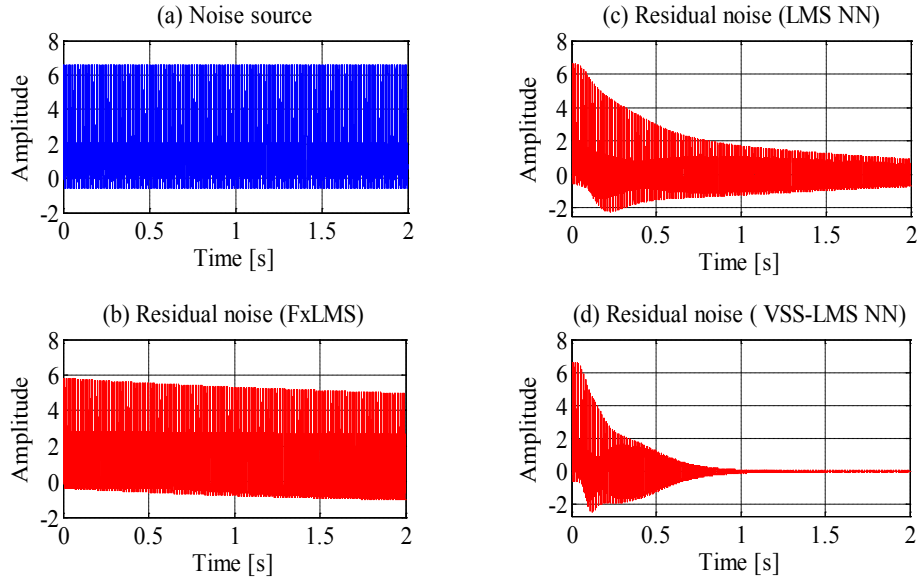


Figure 12. Simulation results for noise with frequencies 100Hz and 200 Hz in time domain. (a) Noise source; (b) Residual noise of linear control with FxLMS; (c) Residual noise of NN control with LMS; (d) Residual noise of NN control with VSS-LMS.

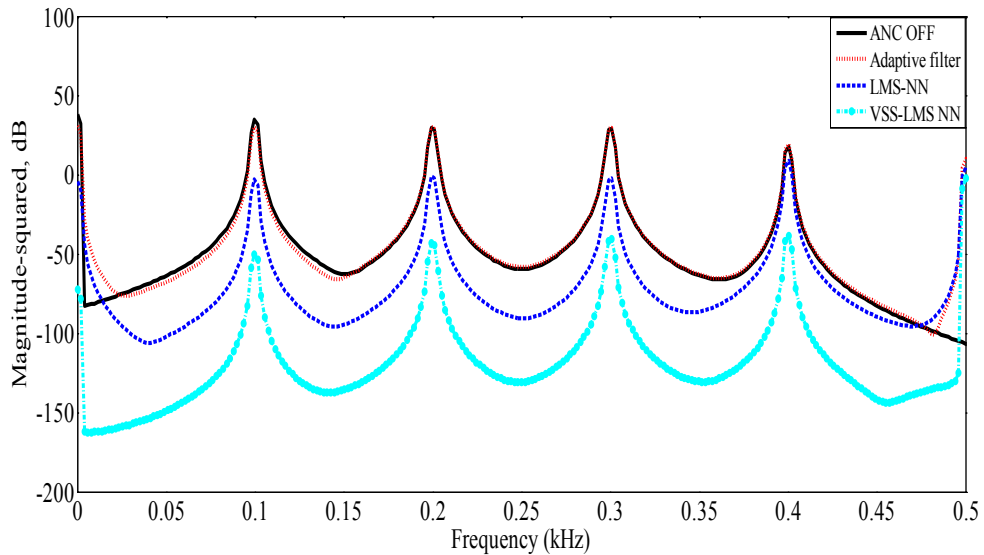


Figure 13. Simulation results for noise frequencies 100Hz and 200 Hz in frequency domain.

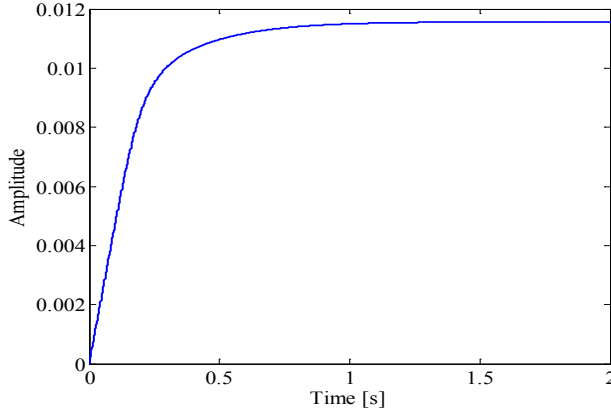


Figure 14. Convergence performance of  $\mu'$ .

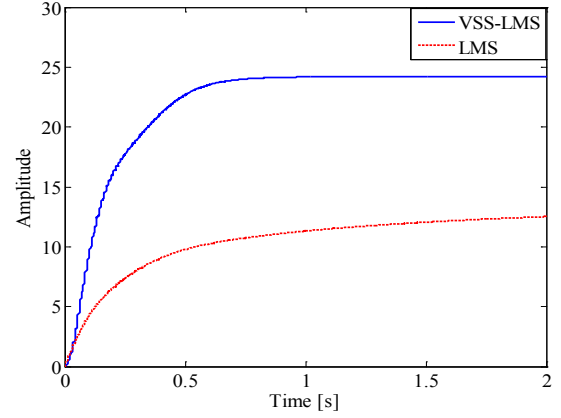


Figure 15. Adaptation of coefficient  $w^{(2)}$  in NN control.

All of above simulations indicate that how the time-varying learning gains can improve the convergence response of the adaptive parameters used in the control, which also help to achieve better noise attenuation. Moreover, neural network can handle nonlinearities encountered in the control system, so that the proposed nonlinear controls with neural network can obtain better control response than linear control with FIR filter and constant step-size.

## 5. Conclusions

In this paper, we introduce a nonlinear neural controller for feedback ANC system, where a neural network is employed as the control to handle complex noise signals and nonlinear dynamics in the primary path and secondary path of ANC systems. In comparison to widely used linear FIR control, this nonlinear control approach can achieve better control performance although it needs increased computational costs. Moreover, a new algorithm is suggested to online update the learning gain (step-size) used in the adaptive laws. This modified adaptive law with a variable step-size is used to estimate the unknown control parameters (e.g. NN weights), which achieves faster convergence response. Specifically, we provide a new theoretical framework to analyze the stability of feedback ANC systems, and also rigorously prove that the online updated step-size and the NN weights can converge to their ideal values. Finally, extensive comparisons to standard linear FIR control with constant step-size are conducted to address the computational complexities and the convergence performance. The effectiveness and the improved performance of the proposed approach are validated by means of numerical simulations.

### Appendix 1- the proof of Theorem 1

**Proof:** We choose a Lyapunov function as

$$V_1(n) = \frac{1}{2} e^2(n) = \frac{1}{2} [d(n) - v(n)]^2 \quad (44)$$

Then the difference  $\Delta V_1(n) = V_1(n+1) - V_1(n)$  of (44) can be calculated as

$$\begin{aligned} \Delta V_1(n) &= \frac{1}{2} [e^2(n+1) - e^2(n)] = \frac{1}{2} [e(n+1) - e(n)][e(n+1) + e(n)] \\ &= \frac{1}{2} \Delta e(n) [2e(n) + \Delta e(n)] \end{aligned} \quad (45)$$

The error  $\Delta e(n)$  used to update the coefficients  $w^{(1)}$  is calculated as



$$\begin{aligned}
\Delta e(n) &= e(n+1) - e(n) = \left[ \frac{\partial e}{\partial w} \right] \Delta w^{(1)}(n) = \left[ \frac{\partial e}{\partial v} \sum_{j=0}^J \frac{\partial v(n)}{\partial y(n-j)} \frac{\partial y(n-j)}{\partial f(n-j)} \frac{\partial f(n-j)}{\partial g(n-j)} \frac{\partial g(n-j)}{\partial w^{(1)}(n)} \right] \Delta w^{(1)}(n) \\
&= \left\{ \sum_{j=0}^J s_j [1 - f^2(n-j)] w^{(2)}(n) \hat{d}(n-j) \right\} \left\{ -\mu_1 e(n) \sum_{j=0}^J s_j [1 - f^2(n-j)] w^{(2)}(n) \hat{d}(n-j) \right\} \\
&= -\mu_1 e(n) \left\| \sum_{j=0}^J s_j [1 - f^2(n-j)] w^{(2)}(n) \hat{d}(n-j) \right\|^2
\end{aligned} \tag{46}$$

Substituting (46) into (45), one can have

$$\begin{aligned}
\Delta V_1(n) &= -\frac{1}{2} \mu_1 e(n) \left\| \sum_{j=0}^J s_j [1 - f^2(n-j)] w^{(2)}(n) \hat{d}(n-j) \right\|^2 \left\{ 2e(n) - \mu_1 e(n) \left\| \sum_{j=0}^J s_j [1 - f^2(n-j)] w^{(2)}(n) \hat{d}(n-j) \right\|^2 \right\} \\
&= -\frac{1}{2} \mu_1 e^2(n) \left\| \sum_{j=0}^J s_j [1 - f^2(n-j)] w^{(2)}(n) \hat{d}(n-j) \right\|^2 \left\{ 2 - \mu_1 \left\| \sum_{j=0}^J s_j [1 - f^2(n-j)] w^{(2)}(n) \hat{d}(n-j) \right\|^2 \right\}
\end{aligned} \tag{47}$$

From (44), it is shown that if the step-size  $\mu_1$  is chosen too large, then the condition

$$2 - \mu_1 \left\| \sum_{j=0}^J s_j [1 - f^2(n-j)] w^{(2)}(n) \hat{d}(n-j) \right\|^2 > 0 \text{ holds, which further implies } \Delta V_1(n) \leq 0. \text{ In this case, } V_1(n) \text{ will}$$

decrease to zero, i.e.  $V_1(n) \rightarrow 0$  for  $n \rightarrow \infty$ . Consider the facts that  $V_1(n) = \frac{1}{2} e^2(n) \geq 0$  and  $V_1(n) = 0$ , according to

Lyapunov theorem, the error convergence condition of  $w^{(1)}(n)$  can be given as

$$0 < \mu_1 < \frac{2}{\left\| \sum_{j=0}^J s_j [1 - f^2(n-j)] w^{(2)}(n) \hat{d}(n-j) \right\|^2} \tag{48}$$

Similar to the above analysis, the convergence condition for the coefficients  $w^{(2)}$  can be given as

$$0 < \mu_2 < \frac{2}{\left\| \sum_{j=0}^J s_j f(n-j) \right\|^2} \tag{49}$$

## Reference

- [1] Gupta S, Ghatak C. Environmental noise assessment and its effect on human health in an urban area. *Int J Environ SCI* 2011;1:1954-64.
- [2] Tint P, Tarmas G, Koppel T, Reinhold K, Kalle S. Vibration and noise caused by lawn maintenance machines in association with risk to health. *Agronomy Research* 2012;10:251-60.
- [3] Gorj-Bandpy M, Azimi M. Passive Techniques for Fan Noise Reduction in New Turbofan Engines: Review. *Journal of Engineering Science and Technology Review* 2013;6:59-61.
- [4] Elliott SJ, Nelson PA. Active noise control. *Signal Processing Magazine, IEEE* 1993;10:12-35.
- [5] Elliot S. *Signal Processing for Active Control*: Academic Press, London, UK; 2001.
- [6] Misol M, Algermissen S, Monner H. Experimental investigation of different active noise control concepts applied to a passenger car equipped with an active windshield. *Journal of Sound and Vibration* 2012;331:2209-19.
- [7] Sun X, Kuo SM, Meng G. Adaptive algorithm for active control of impulsive noise. *Journal of Sound and Vibration* 2006;291:516-22.
- [8] Yang Z, Gu Z, Tu J, Dong G, Wang Y. Numerical analysis and passive control of a car side window buffeting noise based on Scale-Adaptive Simulation. *Applied Acoustics* 2014;79:23-34.
- [9] Zhang L, Qiu X. Causality study on a feedforward active noise control headset with different noise coming directions in free field. *Applied Acoustics* 2014;80:36-44.
- [10] Zeng J, De Callafon RA. Recursive filter estimation for feedforward noise cancellation with acoustic coupling. *Journal of sound and vibration* 2006;291:1061-79.

- [11] Peretti P, Cecchi S, Romoli L, Piazza F. Adaptive Feedback Active Noise Control for Yacht Environments. *IEEE Transactions on Control Systems Technology* 2014;22:1063-6536.
- [12] Wu L, Qiu X, Guo Y. A simplified adaptive feedback active noise control system. *Applied Acoustics* 2014;81:40-6.
- [13] Behera SB, Das DP, Rout NK. Nonlinear feedback active noise control for broadband chaotic noise. *Applied Soft Computing* 2014;15:80-7.
- [14] Song Y, Gong Y, Kuo SM. A robust hybrid feedback active noise cancellation headset. *Speech and Audio Processing, IEEE Transactions on* 2005;13:607-17.
- [15] F. A. González RR, G. R. Molina and G. Parlanti. Adaptive Narrowband ANC, Design and Implementation Issues. *Latin America Transactions, IEEE (Revista IEEE America Latina)* July 2011;9:438 - 44
- [16] Das DP, Panda G. Active mitigation of nonlinear noise processes using a novel filtered-s LMS algorithm. *Speech and Audio Processing, IEEE Transactions on* 2004;12:313-22.
- [17] Zhao H, Zeng X, Zhang J. Adaptive reduced feedback FLNN filter for active control of nonlinear noise processes. *Signal Processing* 2010;90:834-47.
- [18] Kuo SM, Wu H-T. Nonlinear adaptive bilinear filters for active noise control systems. *Circuits and Systems I: Regular Papers, IEEE Transactions on* 2005;52:617-24.
- [19] Zhao H, Zeng X, He Z, Li T. Adaptive RSOV filter using the FELMS algorithm for nonlinear active noise control systems. *Mechanical Systems and Signal Processing* 2013;34:378-92.
- [20] Park J-W, Venayagamoorthy GK, Harley RG. MLP/RBF neural-networks-based online global model identification of synchronous generator. *Industrial Electronics, IEEE Transactions on* 2005;52:1685-95.
- [21] Zhang X, Ren X, Na J, Zhang B, Huang H. Adaptive nonlinear neuro-controller with an integrated evaluation algorithm for nonlinear active noise systems. *Journal of Sound and Vibration* 2010;329:5005-16.
- [22] Montazeri A, Poshtan J. A new adaptive recursive RLS-based fast-array IIR filter for active noise and vibration control systems. *Signal Processing* 2011;91:98-113.
- [23] Douglas SC. Fast implementations of the filtered-X LMS and LMS algorithms for multichannel active noise control. *IEEE Transactions on Speech and Audio Processing*, 1999;7:454-65.
- [24] Tang XL, Lee CM. Time-frequency-domain filtered-x LMS algorithm for active noise control. *Journal of Sound and Vibration* 2012;331:5002-11.
- [25] Kwong RH, Johnston EW. A variable step size LMS algorithm. *IEEE Transactions on Signal Processing* 1992;40:1633-42.
- [26] Aboulnasr T, Mayyas K. A robust variable step-size LMS-type algorithm: analysis and simulations. *IEEE Transactions on Signal Processing*, 1997;45:631-9.
- [27] Huang B, Xiao Y, Sun J, Wei G. A variable step-size FXLMS algorithm for narrowband active noise control. *IEEE Transactions on Audio, Speech, and Language Processing*, 2013;21:301-12.
- [28] Chang DC, Chu FT. Feedforward Active Noise Control With a New Variable Tap-Length and Step-Size Filtered-X LMS Algorithm. *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, 2014;22:542-55.
- [29] Wang LV, Gan W-S, Khong AW, Kuo SM. Convergence analysis of narrowband feedback active noise control system with imperfect secondary path estimation. *Audio, Speech, and Language Processing, IEEE Transactions on* 2013;21:2403-11.
- [30] Kuo SM, Kong X, Gan WS. Applications of adaptive feedback active noise control system. *Control Systems Technology, IEEE Transactions on* 2003;11:216-20.
- [31] Selmic RR, Lewis FL. Neural-network approximation of piecewise continuous functions: application to friction compensation. *Neural Networks, IEEE Transactions on* 2002;13:745-51.
- [32] Arandiga F, Cohen A, Donat R, Dyn N, Matei B. Approximation of piecewise smooth functions and images by edge-adapted (ENO-EA) nonlinear multiresolution techniques. *Applied and Computational Harmonic Analysis* 2008;24:225-50.
- [33] Eriksson L, Allie M. Use of random noise for on - line transducer modeling in an adaptive active attenuation system. *The Journal of the Acoustical Society of America* 1989;85:797-802.
- [34] Kuo SM, Vijayan D. Optimized secondary path modeling technique for active noise control systems. 1994 *IEEE Asia-Pacific Conference on Circuits and Systems, (APCCAS'94): IEEE*; 1994. p. 370-375.
- [35] Li Tan, Chen Dong, Sidan Du. On implementation of adaptive bilinear filters for nonlinear active noise control. *Applied Acoustics* 2016;106: 122–128.